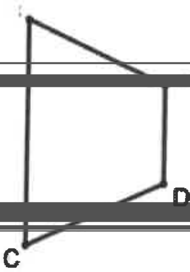
2023 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet [redacted]

13. When $x^4 - 33$ is divided by $x - 3$, find the remainder.

14. If $43°20'$ were expressed in terms of degrees [illegible]



Name: ______________________ Team Code: ________

2023 John O'Bryan Mathematical Competition
Freshman/Sophomore Individual Test

Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each [illegible]

1. ______________________

2. ______________________

3. ______________________

4.

5.

6.

7.

8.

9.

10.

11. ______________________

12. ______________________

13.

14.

15.

16.

17.

18.

19.

20.

Name: ANSWERS

Team Code:

2023 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of ...

4. 1 / 3 — Must be this fraction

6. 171 / 29 — Must be this fraction

7. 11 / 18 — Must be this fraction

8. 74

9. 66

10. 405

13.

14. 509 / 120 — Must be this fraction

15.

16. 5

17. 4.4 — Must be this decimal

18. 314.8 — Must be this decimal

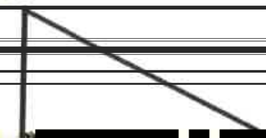
19. 73

20. 221 / 30 — Must be this fraction

Awards Lists and Solutions to the Team Competition may be found at http://math.nku.edu/job

2023 John O'Bryan Mathematical Competition

[redacted]



Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly in the correct blanks on the answer sheet and in simplest [redacted]

12. An apartment rental company has 3000 apartments available, and 1900 are presently rented at $950 per month. The company has decided it will only raise or lower the rent per month on all apartments by integral increments of $40. A survey has shown that, so long as there are apartments available, for each

[redacted]

Name: **Team Code:**

2023 John O'Bryan Mathematical C

[redacted]

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

Name: ______ANSWERS______ Team Code: ________

2023 John O'Bryan Mathematical Competition

2. 13 / 15 — Must be this fraction.

3. 1.2 — Must be this decimal.

4. 1 / 6 — Must be this fraction.

5. – 4.8 — Must be this decimal.

6. 10

7. π

8. 1 / 2 — Must be this fraction.

9. 13.13 — Must be this decimal.

10. 17

12. 870

13. 11

14. 32

15. 32 / 53 — Must be this fraction.

16. 5

17. 172

18. 1073741823

19. 4.00

20. 27719

Names: **School:**

2023 John O'Bryan Mathematical Competition
Two-Person Speed Event

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

Calculators are not allowed to be used on the first four questions!

This competition consists of eight [illegible] competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

	SCORE
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	

SCORE

T1.	
T2.	

Names: **School:**

2023 John O'Bryan Mathematical Competition

[redacted]

2.

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the

3. 75

4. −21

[redacted]

5. 63

SCORE

Calculators are not allowed to be used on the first four questions!

1. 30

6. 3151

7. 6.65 **Must be this decimal**

This competition consists of eight competitive rounds. Correct

8. 1

SCORE

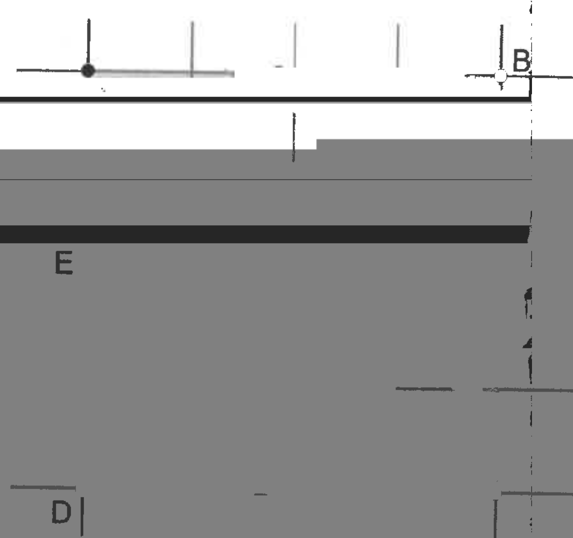
T1. 784

T2. [redacted] **Must be this decimal**

2023 John O'Bryan Mathematics Competition
5-Person Team Test

Abbreviated Instructions: Answer each of the following questions using separate sheet(s) of paper. Questions

[redacted]



[redacted]

Place your team code in the

[redacted]

4. Complete each of the following. **Remember for all problems you must show or explain** [redacted]

a. Determine the positive integer c for which $\frac{1}{4} - \frac{1}{c} = \frac{1}{6}$

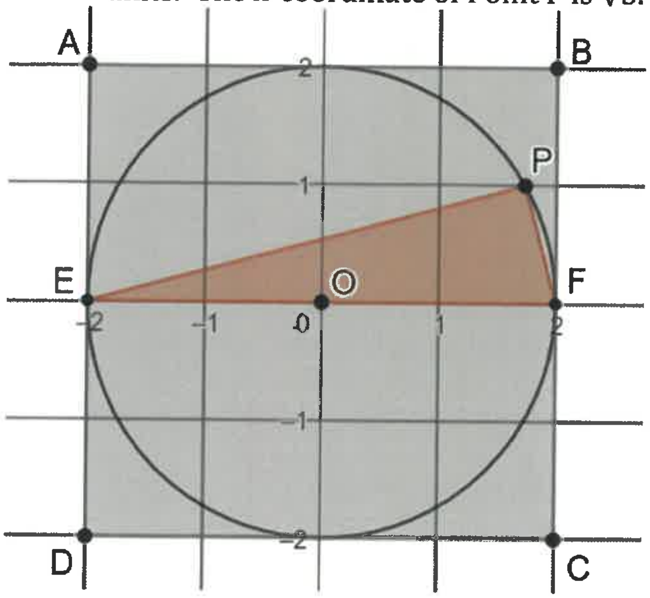
b. Determine the number of pairs of positive integers (d, e) for which $\frac{1}{d} - \frac{1}{e} = \frac{1}{12}$

[redacted]

$\neq$

John O'Bryan 2023 Team Test Key

1. In the figure below, quadrilateral ABCD is square and inscribed circle O has a radius of 2 units. The x-coordinate of Point P is $\sqrt{3}$.



a. Find the area of triangle EPF

The equation of the circle is $x^2 + y^2 = 4$. If the x-coordinate of P is $\sqrt{3}$, then $(\sqrt{3})^2 + y^2 = 4$. So, $y = 1$. With a base of 4 and altitude of 1, the area of triangle EPF is $(1/2)(4)(1) = 2$ square units

b. Find the sum of the squares of the distances from point P to each vertex of the square [redacted]

Using the distance formula:

$PA^2 = \left(\sqrt{3} - (-2)\right)^2 + (1 - 2)^2 = 8 + 4\sqrt{3}$

$PB^2 = \left(\sqrt{3} - 2\right)^2 + (1 - 2)^2 = 8 - 4\sqrt{3}$

$PC^2 = \left(\sqrt{3} - 2\right)^2 + (1 - (-2))^2 = 16 - 4\sqrt{3}$

$PD^2 = \left(\sqrt{3} - (-2)\right)^2 + (1 - (-2))^2 = 16 + 4\sqrt{3}$

So, $PA^2 + PB^2 + PC^2 + PD^2 = 48$

c. Does the answer to part (b) depend upon the location of point P or will you obtain the

No, the sum of the squares remains the same (48) for any point on the circle. To show this is true for any point on the to[illegible] half of the circle (the case for points [illegible] lower half

2. On day 1, Paris writes the number 1.
On day 2, she writes the numbers 2 and 3.
On day 3, she writes the numbers 4, 5, and 6.
Paris continues writing numbers in this way, writing N numbers on the Nth day.

a. What is the largest number that Paris writes on the 20^{th} day? Show or explain how you obtained your answer.

From the pattern, the largest number written on day N equals the sum of the first n natural numbers = $\frac{N(N+1)}{2}$. For instance, the largest number written on day 1 = $\frac{1(1+1)}{2}$ = 1, the largest number written on day 2 = $2(2+1)$ [illegible] $3(3+1)$ [illegible]

b. [redacted]

[illegible]

c. What is the sum of the numbers that Paris writes on the Nth day? Justify your answer.

[illegible]

4. a. Determine the positive integer c for which $\frac{1}{4} - \frac{1}{c} = \frac{1}{6}$

Multiply the equation by 12c, so that $3c - 12 = 2c$. So, $c = 12$

b. Determine the number of pairs of positive integers (d, e) for which $\frac{1}{d} - \frac{1}{e} = \frac{1}{12}$

d-12	e+12	d	e
−1	144	11	132
−2	72	10	60
−3	48	9	36
−4	36	8	24
−6	24	6	12
−8	18	4	6
−9	16	3	4

Multiply the equation by 12de to obtain $12e - 12d = de$.

So:

$0 = de + 12d - 12e$

$0 = d(e + 12) - 12e$

$-144 = d(e + 12) - 12e - 144$

$-144 = d(e + 12) - 12(e + 12) = (d \quad 12)(e + 12)$

Since $e > 0$, $e + 12 > 0$ and $d - 12 < 0$ (since the product

c. Show that for every prime number p, there are at least two pairs (r, s) of positive integers for which $\frac{1}{r} - \frac{1}{s} = \frac{1}{p^2}$

T(4,1) = 4. Note, the result is the same if the point is added on the boundary of an interior triangle.

If an interior point is added to one of the T(4,1) triangles, T(4,1) is reduced by one

c. Determine the value of n for which T(n,n) = 1000

6. Define the function $f(x) = \frac{x}{x-1}$ for $x \neq 1$.

a. Determine all real numbers $r \neq 1$ for which $f(r) = r$

If $f(r) = r$, then $\frac{r}{r-1} = r$

Since $r \neq 1$, $r = r(r - 1)$ or $r = r^2 - r$.
Thus, $r^2 - 2r = r(r - 2) = 0$, so $r = 0$ or $r = 2$.

b. Find $f(f(x))$ [illegible]

Since $f(x) = \frac{x}{x-1}$, $f(f(x)) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-(x-1)} = x$

c. Suppose k is a real number and define $g(x) = \frac{2x}{x+k}$ for $x \neq -k$. Determine all real values of k for which $g(g(x)) = x$, where $g(x) \neq -k$.

If $x \neq -k$, then $g(x)$ is defined for all other real values.

If $x \neq -k$ and $g(x) \neq -k$, then $g(g(x)) = \frac{2\left(\frac{2x}{x+k}\right)}{\frac{2x}{x+k}+k}$ is defined for all other real values.

Let $g(g(x)) = \frac{2\left(\frac{2x}{x+k}\right)}{\frac{2x}{x+k}+k} = x$.

Then, $\frac{4x}{2x+k(x+k)} = x$

So:
$4x = x(2x + k(x + k))$
$4x = x(2x + kx + k^2)$
$4x = 2x^2 + kx^2 + k^2x$
$0 = (2 + k)x^2 + (k^2 - 4)x$
$0 = (k + 2)[x2 + (k - 2)x]$

If **$k = -2$**, $\frac{4x}{2x+k(x+k)} = x$ for all values of x for which the function is defined (i.e., $x \neq -k$, and $g(x) \neq -k$)